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Universal/Variable Life Insurance Policy Purchase Decisions

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Abstract

Universal/variable life insurance combines the tax advantages of cash value life insurance with investment in money market and equity market funds. Despite upfront loads on universal/variable life policies, this tax treatment often generates a greater after tax return on these policies than similar alternative investment strategies. This paper provides a method for calculating relative after tax proceeds on universal/variable life and comparable alternative investment strategies, illustrates minimum holding periods for a number of parameter values, indicates a method to determine optimal premium levels for a universal/variable life policy, and discusses the effect of tax reform on the attractiveness of universal/variable life insurance.

Introduction

Universal life insurance, introduced in 1979, and universal/variable (also known as flexible premium variable) life insurance, approved by the Securities and Exchange Commission in November, 1984, provide the tax sheltered treatment of investment earnings inherent in cash value life insurance policies with the insured retaining the investment risk. In both policies the investment medium is similar to that offered to non-insurance purchasers. In universal life policies, the cash value is invested in a money market fund; for universal/variable life insurance policies, the cash value can be invested in any of a variety of alternatives generally including stock market funds, long term bond funds, and money market funds.

The typical universal life policy includes an expense loading, either flat rate or as a percentage of premiums, and an insurance charge based on the insured's mortality risk, with the remainder invested in a cash value account that earns a money market rate of interest [17]. Premiums are not predetermined; within fairly wide limits the insured has flexibility in the amount of premiums paid. Since the insured retains the investment risk, changes in money market interest rates directly affect the return on the policy's cash value. Death benefits generally equal the initial face value of the policy plus any cash value, although some policies provide only the initial face value as the death benefit.

Universal/variable life insurance policies, currently being introduced into the market, are similar in structure to universal life with a wider array of investment options. They differ from current variable

life policies considerably, notably in the discretionary premium levels, the distinct expense loadings, and the term insurance rate structure for the mortality risk. All investment choices, equity funds, bond funds, and specialized investment pools, will be similar to investments generally available to the public outside of a life insurance policy, although competing investments do not have the same tax treatment. As universal/variable life insurance encompasses the basic features of universal life, with additional investment options, the term universal/variable will be used to apply to both policy types.

The tax advantage of life insurance policies becomes increasingly important the longer the policy is kept in force. Taxes on investment earnings are deferred until the cash value is withdrawn. If the policy is surrendered for the cash value, only the excess of cash value over all premiums paid is taxable; investment earnings that are offset by expense loadings and insurance costs are never taxed. If the cash value is paid as part of the death benefit, no income tax is payable on any investment earnings. Since the tax advantage of life insurance policies increases with the holding period of the policy, there is generally a specific holding period after which investment in the universal/variable life insurance policy dominates a similar investment strategy without the life insurance tax advantage. Policies held for shorter periods of time underperform alternative investments, primarily due to the expense loading inherent in the life insurance policies. In this paper, the minimum holding period for which the universal/variable life insurance policy dominates the alternative investment strategy is calculated for the range of policy conditions, investment choices and rates of return obtainable.

Literature Review

Prior to the development of life insurance policies that left the investment risk with the insured, analysis of life insurance purchase decisions and competing investment alternatives (buy term and invest the difference) compared an interest rate guarantee against a hypothetical investment return [4, 6, 10, 12 pp. 135-45, 13, and 15]. Variations on investment rates of return affected one side of the equation only. More recently, Myers and Pritchett [14] examined the rate of return over 20 years on differential premiums between those paid on participating and nonparticipating policies for policies issued in 1959. The achieved rate of return depended heavily on the length of time the policy was kept in force. For policies kept in force for the full 20 year period, returns exceeded those available on competing investments.

Another study comparing investment options between a tax advantaged insurance product, in this case an annuity, and alternative investments was performed by Adelman and Dorfman [1]. Although this study ignored capital gains treatment of equity investment alternatives, the effect of different tax levels was measured. Again the holding period proved to be an important factor in evaluating the more advantageous investment.

Analysis of life insurance purchase decisions for universal/variable life includes the same rate of return forecast on both the life insurance policy and the competing investment alternative. As Belth [3] notes, insurers' rates of return on the savings component of universal life insurance differ depending on whether expense loadings are treated as a protection element or a savings element.

If the expense loading is regarded as a savings element, the rate of return may be negative, whereas if the expense loading is allocated to the protection element of the policy, the rate of return could be quite high relative to alternative investments.

Investment Value Determination

The value of an investment in a front loaded universal/variable life insurance policy that has a death benefit equal to the initial face value plus the cash value can be determined as follows:

$$UVL = \begin{cases} \sum_{i=1}^{n} ((1-e)P_i - g \cdot F \cdot C_{x+i-1})(1+r+d)^{n-i+1} & \text{if } UVL \leq \sum_{i=1}^{n} P_i \\ (1-t)\left[\sum_{i=1}^{n} ((1-e)P_i - g \cdot F \cdot C_{x+i-1})(1+r+d)^{n-i+1} - \sum_{i=1}^{n} P_i\right] + \sum_{i=1}^{n} P_i & \text{otherwise} \end{cases}$$

where P_i = premium paid in year i

n = number of years the policy is kept in force (holding period)

e = front end expense loading (as percentage of premium)

g = index of competitiveness of term insurance through universal life policy

F = face value of the policy

C_x = cost of term insurance for policyholder age x

r = annual rate of return for comparable investment fund

d = differential between policy interest rate and comparable investment fund rate

t = marginal tax rate of insured

The amount invested in the cash value each year is the premium less an expense loading, e, and less the cost of insurance. The cash value earns a rate of return, r+d, that tracks below, at or above, comparable

investment rates of return. The investment earnings are not taxed until the policy is surrendered. If, at that time, the cash value does not exceed the total premiums paid, no income tax liability exists. If the cash value does exceed the premiums paid, the excess is taxed at the insured's current marginal tax rate. When the policyholder elects to invest the cash value at money market fund rates, this life insurance purchase decision can be compared with a strategy of buying term insurance and investing the remaining sum in a money market fund. The value of this investment would be:

$$BTID_M = \sum_{i=1}^{n} (P_i - F \cdot C_{x+i-1})(1 + (1-t)r)^{n-i+1}$$

The investment proceeds are taxed each year under this alternative, reducing the current yield. No expense loading is deducted from the amount to be invested. The cost of insurance is simply the lowest priced coverage available in a renewable term policy. Note that this can be higher than, equal to or lower than the rate charged in the universal life policy depending on whether, g, the index of competitiveness of the insurance costs through the universal life policy, is less than, equal to, or greater than one. The rate of return is simply the standard money market fund rate.

The relative values of UVL and $BTID_M$ depend on the parameters. An example of the after tax surrender values for a specific selection of parameters is illustrated on Table 1.

Insert Table 1 here

The same analysis can be performed assuming the policyholder elects investment in equity funds, which have different tax treatment from

money market funds. In a stock market fund realized short term capital gains are taxed currently at ordinary income tax rates. Realized long term capital gains are taxed currently, but only 40 percent of the gain is taxable. Dividends are taxed currently at ordinary income tax rates with a $100 per taxpayer exclusion for dividends of domestic corporations. Unrealized gains are not taxed until shares of the fund are sold; any gains thus realized may be subject to long term capital gains treatment depending on the holding period. For equity gains in a universal/variable life insurance policy, no long term capital gains treatment applies; taxes are deferred, but any gain over premiums is taxed at ordinary income rates regardless of the holding period.

The alternative stock fund investment strategy includes tax advantages not found in a money market fund investment. The value of this alternative is:

$$BTID_S = \sum_{i=1}^{n} (P_i - F \cdot C_{x+i-1})(A^{n-i+1} - .4t(A^{n-i+1} - (1 + ([s(1 - t)r + \ell(1 - .4t)r] \sum_{k=i}^{n} A^{n-k}))))$$

where $A = (1 + r - str - .4\ell tr)$

s = proportion of r produced by realized short term capital gains and dividends

ℓ = proportion of r produced by realized long term capital gains

If the stock mutual fund did not generate any realized short or long term capital gains or dividends, no taxes would be payable until the shares were sold. If realized gains or dividends were generated, the investor has the option of reinvesting those amounts or receiving them

in a cash distribution. Since the taxes owed would always be less than the cash distribution, the investor can pay the taxes out of the distribution and reinvest the remainder back in the money market fund. Thus, the basis in the fund would reduce only by any taxes paid and the investor would not retain any excess cash. Under this procedure, no shares would have to be sold to pay taxes. This situation is preferred because any sale of shares would involve capital gains taxes on any unrealized (by the fund) gains, which would result in additional taxes payable.

An example of the relative after tax surrender values for a univeral/variable life insurance policy and an alternative stock mutual fund investment strategy is illustrated on Table 2.

Insert Table 2 here

Parameter Values

The objective of this research project is to analyze the necessary holding periods for which the universal/variable life insurance policies dominate similar investment strategies in money market funds and equity funds outside of life insurance policies for the range of parameters available. The values of the ten parameters used to evaluate a universal/variable life insurance policy vary significantly depending upon the potential policyholder and the specific policy. The effect of varying these values is examined in this paper. Determination of the standard values and the ranges used are discussed in this section.

The rate of return, r, used in this analysis is the money market interest rate or the equity fund total rate of return. This value

indicates the rate payable on a competing investment alternative; it could be considered either the average rate paid by money market or equity funds or the rate paid by a particular fund. The relevant rate of return is that experienced after the investment choice, universal/variable life or buy term, is made. Thus, it is a forecasted value, not a historical value, that indicates the preferred investment. As such, a range of values of r should be examined by a potential policyholder. Since money market funds became popular in the late 1970s, rates of return have ranged from 5 to 17 percent [7, p. 226]. Short term interest rates prior to 1950 ranged in the 2-4 percent level. Investments in common stocks have historically provided a higher rate of return than short term interest rates for any extended period. Over the period 1926-1976 equity investments produced a geometric average return of 9.2 percent versus a short term bill average return of 2.4 percent [8]. Over the ten year period 1974 through 1984 the average equity mutual fund generated an annual return of 17.5 percent [18]. For this analysis, the rate of return is allowed to range from 4 to 20 percent. Universal/variable life insurance policies generally provide minimum guaranteed rates of around 4 percent. For forecasted rates of return below this level, it is unlikely that anyone would consider investment in universal/variable life insurance. The standard rate of return is 10 percent for money market investments and 15 percent for equity investments.

The tax rate, t, is the individual's marginal tax rate each year under the buy term strategy or when the cash value is withdrawn under the life insurance strategies. The tax rate is assumed to be constant,

although tax rates do change over the life cycle of a policyholder and as government revenue needs vary. Although many insurers illustrate the value of insurance products assuming a lower tax rate after retirement, this procedure is not included in this analysis. The taxability of pensions, individual retirement account withdrawals, and one-half of social security benefits, combined with uncertainty over future tax rate levels, makes a reduction in marginal tax rates an uncertain proposition. Also, the variety of potential patterns of changes in tax rates over time suggests use of a simple constant tax rate as a reasonable approximation. In this analysis the tax rate ranges from 0 to 50 percent, with the standard rate set at 40 percent.

Expense loadings on universal/variable life insurance policies take a variety of forms, including a flat fee per policy, a change based on the amount of coverage, a percentage of the investment, or a combination of these changes. In some cases expense loadings are constant over the life of the contract whereas other policies reduce expenses after the first year [17]. In this analysis the expense loading, e, is determined as a constant percentage of annual investments. This value ranges from 2 to 20 percent, with a standard value of 6 percent.

The interest rate differential, d, indicates how the interest rate credited on the universal/variable life policy compares with rates of return available on comparable investments. Some universal life insurance policies have an interest rate that is tied to a short term interest rate level, such as 90 day Treasury bills, but for a majority of the policies interest rates are established by the insurer. The policyholder in these situations has no guarantee that the company will

not alter past patterns of interest levels, but any change would affect all policyholders. For other universal/variable life insurance policies the rate of return earned on cash values is not controllable by the insurer, but depends on short term bond or equity investment performance. Administrative expenses and investment policy may generate a differential between the return earned by the insurance fund and other public funds with similar risk characteristics. After these policies have established a track record, investment performance could be analyzed to project a differential value. Given the current lack of an investment record for universal/variable life insurance funds, expense loadings could be compared to project a difference. The differential used in this analysis is a percentage point difference between the universal/variable life rate of return and the comparable fund rate of return. The differential is constant over the life of the policy and values range from negative 4, in which the insurer credits the cash value with a rate of return 4 points below comparable fund rates, to a positive 4. The standard differential is zero.

Portfolio turnover also affects the relative attractiveness of investment in a universal/variable life insurance policy. Any gains realized by the investment fund in this policy are tax deferred until the policy is surrendered and then taxed at ordinary income rates to the extent cash value exceeds premiums paid. In the competing equity investment, short term capital gains are taxed currently at ordinary income tax rates, dividends are taxed currently at ordinary income tax rates after the $100 dividend exclusion, and 40 percent of the long term capital gains are taxed currently at ordinary income tax rates.

Stock funds have a wide range of portfolio turnover rates. A sample of funds examined indicated values of 20 percent to in excess of 200 percent. Higher turnover and dividend yield increases the current taxation on the competing investment strategy and improves the position of universal/variable life. Current tax law applies long term capital gains treatment to securities held longer than six months.

For this analysis, the standard rate of return on stock investments is 15 percent. Dividends are likely to account for 5 percentage points and capital gains 10 percentage points, but this relationship will vary depending on the objective of the fund. If realized short term capital gains account for 25 percent of the capital gains and realized long term capital gains 45 percent, then portfolio turnover would be 70 percent, a fairly typical value. The total taxable gain for the mutual fund holder would be:

	Percentage Points	Percent of Total Return
Taxed at ordinary income rates		
Dividends	5	33.33
Realized short term gains	2.5	16.67
subtotal	7.5	50.00
Taxed at capital gains rates		
Realized long term gains	4.5	30.00

The standard values used in this analysis are .50 for s, the proportion of r produced by realized short term capital gains and dividends, and .30 for ℓ, the proportion of r produced by realized long term capital gains. The remaining proportion of r is deferred until the mutual fund is sold. Other values of s and ℓ demonstrated in this research are 0 and 0, if all gains were deferred, 0 and 100, if all gains were realized

long term gains, 100 and 0, if all gains were realized short term gains, and 20 and 30.

The available capital per year, P, is the amount the policyholder wants to invest in either an insurance policy or the buy term and invest the difference strategy. One advantage of the new life insurance policies is the flexibility the policyholder has with regard to premium payments. Within fairly large limits the policyholder can select any investment level and alter the amount at will. Generally the minimum allowed investment is the amount necessary to cover mortality costs, although some policies allow no payment if the cash value is large enough so that mortality costs can be paid by a reduction in cash value. The maximum contribution level is determined by the Tax Equity and Fiscal Responsibility tax law of 1982 that restricts the cash value to an age based percentage of the death benefit. For a policyholder age 40 or less, the death benefit must equal or exceed 140 percent of the cash value in a universal life policy; for insureds over 40, the percentage reduces by one percentage point each year until age 75. Insureds age 75 or over must have a death benefit at least 105 percent of the cash value [5]. For this analysis annual investment levels are assumed constant throughout the policy term; values of $250 to $25,000 are displayed. The standard premium level is $1000.

The face value of the life insurance policy, F, is the amount of coverage initially purchased. This analysis follows the standard practice of determining the death benefit by summing the policy face and the cash value. Thus, the mortality cost is based on a constant amount of coverage. Examples of face values from $25,000 to $5,000,000 are shown. The standard face value is $100,000.

The initial age of the policyholder, x, is used to determine the mortality cost in the life policy and the cost of term insurance in the buy term strategy. Term insurance rates are calculated at one-half the mortality rate shown on the 1980 Commissions Standard Ordinary Mortality Table for males [2]. The CSO Table represents conservative assumptions; current market conditions make term insurance readily available at the assumed rate level. The policyholder's age ranges from 20 to 65 in this study, with 35 used as the standard.

Prior to making the decision of whether to buy a life insurance policy or to buy term insurance and invest the difference in a money market or stock fund, the prospective policyholder would know the face value of the policy desired (F) and his or her age (x) and current tax rate (t). These values do not depend on the insurer or the policy. Also, for each life insurance policy considered, the individual can determine the expense loading (e), how the rates compare with basic term insurance rates (g), and any differential between similar investments and the interest rates credited for the policy (d). The decisionmaker must estimate future rates of return (r), the tax classification for earnings in comparable stock funds under the equity investment option (s and ℓ), and decide the amount to invest (P).

Holding Period Determination

A number of tables are included in this paper that show how many years a universal/variable life insurance policy must be kept in force before that purchase dominates a buy term and invest the difference strategy. The program used to determine the necessary holding periods requires that the universal/variable life insurance death benefit

exceed the cash value by 40 percent regardless of the policyholder's age and requires that the universal/variable life policy dominate withing 30 years. Tables 4 through 11 all follow a similar format, with all parameters except two held constant. A matrix shows the year that the insurance policy dominates investment in either a money market fund or stock fund for the values of the variables listed in that row and column. The invested capital per year varies as shown in the column headings and another variable is changed for each row. The standard parameter values are shown on Table 3.

Insert Table 3 here

Table 4 shows the length of time necessary to hold a universal/variable life insurance policy for it to dominate a buy term and invest the difference strategy under varying rates of return, investment choices and amounts of capital. Rates of return vary from 4 to 20 percent and the annual investment ranges from $250 to $25,000. For a $250 annual investment, as shown in columns 1 and 2, the premium is insufficient to pay expenses and mortality costs over the period necessary for universal/variable life to dominate either investment strategy if the rate of return is 4 percent. For a 6 percent rate of return, universal/variable life is the preferred investment only for a money market fund investment and if the policy is kept in force for ten years or more. For the stock investment choice, the premium is insufficient to pay expenses and mortality costs long enough for the universal/variable life policy to dominate. As the rate of return increases, the necessary holding period under both investment choices reduces; the

value of universal/variable life insurance as a tax shelter increases as the deferred investment earnings increase. For a 20 percent rate of return, universal/variable life invested in the money market option dominates after three years and investment in the stock fund option dominates in four years.

Insert Table 4 here

Columns 3 and 4 show the years universal/variable life dominates for $500 annual investments. For each value of r, a shorter necessary holding period occurs with this greater investment. The amount saved after paying for insurance in the buy term strategy and the cash value inputs after deducting the expense loading and mortality costs in the universal/variable life policy are larger. The expense loading is proportional to the amount invested, but the mortality costs are fixed. Thus, more capital is available for investment giving the tax shelter aspects of universal/variable life insurance more of a advantage. Similarly, increasing the annual investment to $1000 again reduces the necessary holding periods. For a 4 percent rate of return universal/variable life dominates after eight years under the money market fund investment, for a 10 percent rate of return it dominates after three years, and for a 20 percent rate of return it dominates after two years. Note, however, that additional increases in the amount of capital invested increase rather than decrease the necessary holding period. For example, for a $2500 annual investment with a 10 percent rate of return, the universal/variable life policy does not dominate until nine years under the money market fund investment.

Life insurance has two elements of tax savings compared to the alternative investment strategy, and the interaction of these savings causes the necessary holding period to decline and then increase as the available investment increases. The first tax advantage is the deferment of taxation until the cash value is withdrawn. Thus, capital that would be paid in taxes under an alternative investment strategy remains to compound investment earnings in the life policy. The second tax advantage is the inclusion of the cost of insurance in the basis of the universal/variable life insurance investment. The policyholder is taxed only on the excess (if any) of the cash value over all premiums paid. The mortality costs, then, are paid in pre-tax dollars in a life insurance policy versus after-tax dollars in the buy term alternative. Since the value of the latter tax advantage is based on the amount and the cost of the coverage purchased, it becomes a less significant factor as larger amounts of capital are invested. Once enough capital is invested to take full use of this tax advantage, additional investments incur only the tax deferment advantage, which cause a lengthening of the necessary holding period. The optimal investment values are determined and discussed in the next section.

For the standard money market fund investment parameter values, the optimal investment amount is $965 for investment in a money market fund. For a $965 annual investment, universal/variable life insurance dominates the buy term and invest the difference in a money market fund strategy in three years by the greatest proportion. This does not mean that the policyholder should invest only $965 in a universal/variable life policy and deposit any remaining investment capital in a money

market fund. If the available capital were $2500, all of this amount should be placed in a universal/variable life insurance policy if the policy will be held for at least nine years. From Table 4, universal/variable life dominates when the rate of return is 10 percent and the annual investment is $2500 after nine years.

For much larger annual investments, the cash value exceeds the allowable percentage of the death benefit before universal/variable life insurance dominates. In these cases, the policy would have to be adjusted by increasing the face value of the policy, lowering or eliminating annual contributions, or withdrawing money from the cash value. For a 4 percent rate of return, $10,000 annual contributions require a policy adjustment before universal/variable life dominates under both investment choices. For $25,000 annual investments, all rates of return shown other than 20 percent for the money market investment would require an adjustment.

For similar rates of return, the stock investment option always takes longer for the universal/variable life policy to dominate than for the money market fund investment choice. This occurs because equity investments outside a life insurance policy provide more favorable tax treatment than money market fund investments. Some gains in the stock fund are deferred and others are taxed at capital gains rates. However, historically stocks have provided greater returns than short term bonds. For Tables 5 through 11, the assumed rate of return for the stock investment alternative is 15 percent versus 10 percent for the money market fund.

Insert Table 5 here

Table 5 shows the results of varying the expense loading and the annual investment. Regardless of the investment level, the necessary holding period increases as the expense loading increases. This loading is deducted from each capital contribution, so the greater the deduction the longer the holding period necessary to recoup this deduction. For a 2 percent expense loading, the universal/variable life policy dominates within one year for capital contributions of $250 to $1000 under either investment choice; for a very low cost, policyholders are able to pay for the mortality risk in untaxed dollars. For larger contributions, longer holding periods are required. For expense loadings as high as 20 percent, universal/variable life still dominates the buy term strategy if the policy is kept in force for 17 to 27 years, for annual investments of $500 to $1000. At the 20 percent expense level annual investments of $250 are not sufficient to cover the mortality costs for the period necessary for universal/variable life to dominate under the money market fund investment, and universal/variable never dominates for the stock investment. At this expense level premiums in excess of $10,000 generate an excessive cash value requiring policy adjustments.

Insert Table 6 here

The results of varying the tax rate and the annual investment are shown in Table 6. The tax shelter aspect of both life insurance policies is obvious from the first row of this table that indicates that life insurance never dominates the buy term strategy for a policyholder in the zero tax bracket. In every case the premium is insuf-

ficient or excessive or the policy does not dominate within the 30 year period used in this analysis. Regardless of the investment amount or choice, universal life insurance never dominates in less than 20 years until the tax rate increases to 15 percent. As tax rates increase the minimum holding periods decline for all levels of investment. The benefit of a tax shelter is greater the higher the tax rate.

One of the elements of recent tax reform proposals (see Tax Reform section) is to lower the maximum tax bracket from 50 percent to 35 percent. This change by itself would not eliminate the benefit of universal/variable life insurance. For annual investments of $250, universal/variable life dominates either investment allocation after six years. For a $500 annual premium, universal/variable life dominates the money market fund investment choice in four years.

On Table 7 the results of varying the index of competitiveness and the annual investment are displayed. For the first row, the cost of insurance through the universal/variable life policy is 40 percent below the rate charged for insurance under a separate term policy. This lower rate is possible if an insurer covered losses on this segment of the policy through expense loadings and income from managing the investment portfolio. In this case the policyholder is obtaining mortality costs at below market rates with pre-tax dollars. This two-fold advantage results in universal/variable life dominating after only one year for annual investments of $1000 or less under either investment allocation. The larger investment values are not as greatly affected by reduced insurance costs. For $25,000 annual contributions policy adjustments would be required before universal/variable life dominates

either investment option regardless of the index of competitiveness. Annual contributions of $5,000 would require a policy adjustment for the stock fund option.

Insert Table 7 here

Insurers are more likely to change rates above competitive levels for mortality risk in these life insurance policies under the assumption that expense loadings are obvious to the consumer but relative term insurance changes are not. Raising the index of competitiveness to 1.2 or higher creates a situation under which $250 annual investments in a universal/variable life policy are not adequate to cover the mortality costs. This is a dramatic change from a one year holding period when the index is 0.9. Raising the index of competitiveness increases the minimum holding period for all amounts shown, having the greatest impact on the smaller contribution levels.

Insert Table 8 here

The impact on the minimum holding period of varying the policyholder's age and the annual investment is displayed on Table 8. The insurance rates used in this analysis are based on the 1980 CSO Mortality Table for males; the change is one-half the tabular mortality rate. Some of the fluctuations in minimum holding periods for ages 20 and 25 are due to the fact that mortality rates in this table experience a local peak at age 21 and then decline through age 28. For all ages other than 21 through 28, insurance rates increase with age. For amounts of capital from $500 to $2500 per year, the minimum holding

period reduces until a certain age and then increases again under either investment selection. For various investment levels the maximum benefit from buying insurance with pre-tax dollars is achieved within the range of ages shown. The larger the annual investment, the older the age (and greater cost of insurance) optimizes this tax benefit. The high term rates for individuals over 65 are even enough to avoid a policy adjustment requirement for a $25,000 annual investment under the money market fund investment.

Insert Table 9 here

Table 9 illustrates the effect of varying the interest rate differential and the annual contribution on the minimum holding period. For interest rates on the universal/variable life policy 4 percentage points below the rate paid by the alternative investment, universal/variable life never dominates. This differential completely offsets the tax advantage of universal/variable life and the expense loading is never offset. For a negative 3 percentage point differential only the $500 investment level ever dominates under the money market fund investment, and this does only for an unreasonably long holding period of 27 years. For increases in the differential, the minimum holding periods decline for all levels of investment, with the greatest impact on the larger sums where the deferment of investment gains is a greater proportion of the tax advantage of universal/variable life.

Insert Table 10 here

The impact on the necessary holding period of varying the policy face and the annual investment is shown on Table 10. Annual investments of $250 are sufficient to purchase face values only up to $100,000. Larger investments can purchase proportionally higher limits. Conversely, large annual investments are excessive for low face values and require policy adjustments. For annual investments of $500 to $25,000, the minimum holding period declines and then rises over the range of face values shown. This behavior is the result of the tax benefit of covering mortality costs with pre-tax dollars and its relative value as a tax advantage depending on capital contributed. For this analysis mortality costs are proportional to the face value, so if the relative value of investment to coverage is constant, the necessary holding period does not change. For example, $500 investments for $50,000 in coverage produces the same three year required holding period as $5000 investments for $500,000 in coverage for investment in the money market fund.

Insert Table 11 here

The effect of altering the proportion of gains taxable currently as short or long term is shown on Table 11. As these variables do not affect the money market fund investment, under which all gains are taxed currently as ordinary income, the values shown indicate when a universal/variable life policy dominates a stock fund investment. When all gains are tax deferred ($s, \ell = 0$), the universal/variable life policy never dominates for annual investments of $250 and $500, and a policy adjustment is required for larger investments. When all gains

are taxed currently at long term capital gains rates, a $250 annual investment dominates in ten years and a $500 annual investment in 30. Policy adjustments are required for all other investment levels. Conversely, when all gains are taxed currently as short term gains, the stock investment becomes equivalent to a money market fund investment, based on the same 15 percent return. The values for that situation are the same as shown on Table 4 for the money market fund investment. The range of values shown on Table 8 indicates the importance of predicting portfolio turnover and dividend income accurately.

Optimal Premium Levels

In the prior sections the annual investment, P, was given, and the necessary holding period, n, for universal/variable life insurance to dominate a similar unbound investment strategy determined. However, if n and the other parameters are fixed and P allowed to vary, an optimal premium level can be calculated. For this analysis the optimal P, denoted as P*, is set at the value for which the ratio of the difference between the cash value in the insurance policy and the alternative money market or stock fund investment account to the premium is maximized. The formulation of this condition for the universal/variable life policy based on the money market investment option is:

$$\text{Maximize: } \frac{UVL - BTID_M}{P}$$

where UVL and $BTID_M$ are defined previously.

UVL is determined by one of two equations, depending on whether it exceeds the total premiums paid. The shift point, P', where

$UVL = \sum_{i=1}^{n} P_i$, can be calculated by setting UVL equal to nP' and solving for P':

$$P' = \frac{g\,F \sum_{i=1}^{n} C_{x+i-1}\,(1+r+d)^{n-i+1}}{((1-e)\sum_{i=1}^{n}(1+r+d)^{n-i+1}) - n}$$

The derivative of $(UVL - BTID_M)/P$ with respect to P depends on whether $P \leq P'$ or $P > P'$.

For $P \leq P'$,

$$\frac{\partial \frac{UVL - BTID_M}{P}}{\partial P} = \frac{g\,F}{P^2} \sum_{i=1}^{n} C_{x+i-1}\,(1+r+d)^{n-i+1}$$

$$- \frac{F}{P^2} \sum_{i=1}^{n} C_{x+i-1}\,(1+(1-t)r)^{n-i+1}$$

For $P > P'$,

$$\frac{\partial \frac{UVL - BTID_M}{P}}{\partial P} = \frac{(1-t)g\,F}{P^2} \sum_{i=1}^{n} C_{x+i-1}(1+r+d)^{n-i+1}$$

$$- \frac{F}{P^2} \sum_{i=1}^{n} C_{x+i-1}\,(1+(1-t)r)^{n-i+1}$$

The three possible sign combinations for these derivatives are:[1]

Combination	$P \leq P'$	$P > P'$	P*
1	+	+	Maximum allowed
2	+	-	P'
3	-	-	Minimum allowed

For combination 1 the difference increases over the entire range of P, so P* would be the maximum contribution allowed by tax regulations. Absent this restriction, P* would be infinite. For combination 2 the

difference is maximized at P', so P* = P'. The optimal investment level can be calculated by solving equation 4. For combination 3 the difference decreases over the entire range of P, so P* would be the minimum allowable value, generally the mortality and expense costs.

The optimal level of investment, P*, does not assure that universal/variable life is the preferred investment. For that condition to hold, n must equal or exceed the level determined earlier in this paper. This technique only maximizes the ratio of the difference in after-tax investment values to annual investment levels.

The same results occur in determining the optimal premium levels for universal/variable life insurance for a stock fund investment. In this case the derivatives of $(UVL - BTID_S)/P$ with respect to P also depend on whether $P \leq P'$ or $P > P'$.

For $P \leq P'$,

$$\frac{\partial \frac{UVL - BTID_S}{P}}{\partial P} = \frac{g\,F}{P^2} \sum_{i=1}^{n} C_{x+i-1}(1+r+d)^{n-i+1}$$

$$- \frac{F}{P^2} \sum_{i=1}^{n} C_{x+i-1}(A^{n-i+1} - .4t(A^{n-i+1} - (1 +$$

$$([S(1-t)r+\ell(1-.4t)r] \sum_{k=i}^{n} A^{n-k})))$$

For $P > P'$,

$$\frac{\partial \frac{UVL - BTID_S}{P}}{\partial P} = \frac{(1-t)g\ F}{P^2} \sum_{i=1}^{n} C_{x+i-1}(1+r+d)^{n-i+1}$$

$$- \frac{F}{P^2} \sum_{i=1}^{n} C_{x+i-1}(A^{n-i+1} - .4t(A^{n-i+1} - (1 +$$

$$([s(1-t)r + \ell(1-.4t)r] \sum_{k=i}^{n} A^{n-k})))$$

The optimal annual investments in a universal/variable life insurance policy based on investment in a money market investment, P_M^*, or stock fund, P_S^*, are shown on Table 12. For each segment, all parameters are held at the standard values with one allowed to vary over the range that produced meaningful holding periods on Tables 4 through 10. The minimum holding period required for the universal/variable life policy to dominate is used as the number of years the policy is kept in force. The optimal premium level is shown for that selection of parameter values.

Insert Table 12 here

For values of the index of competitiveness, g, of .6 or .8, the universal/variable life policy dominates either alternative investment in one year. The optimal premium level is $106, which is the minimum value possible to avoid insufficient premium. Conversely, for g of 1.4, the universal/variable life policy dominates a money market fund investment in 11 years and a stock fund investment in 19 years. The maximum

allowable investment without requiring a policy adjustment is the optimal investment, or \$15,880 for P_M^* and \$2,870 for $_SP^*$.

For varying values of d, the interest rate differential, P_S^* is consistently less than P_M^*. This relationship occurs because either the stock fund investment is being made for more years, and therefore a lower annual investment is needed, or for those cases where the number of years is the same, the rate of return on the stock fund generates greater investment earnings than the money market fund requiring lower investment sums. Similar relationships hold for varying r, the rate of return, e, the expense loading, and t, the tax rate.

For the standard assumptions the optimal premium level for the money market investment option is \$965. This value is remarkably close to the average universal life premium per policy paid in 1984, which was \$978 [9]. The correspondence of these values is likely to be at least partially coincidental. The average universal policy size was \$82,000, compared with the assumed \$100,000, and the median quoted interest rates ranged from 10.5 to 11.2 percent throughout the year, compared with the assumed 10 percent. No information on the average tax rate or age of the universal life policyholders is available to compare with the assumptions.

Tax Reform

Within the last few months a number of tax reform proposals have been made that could dramatically affect the taxation of life insurance and alternative investments. The Treasury Department's Tax Proposal [16] issued in November, 1984, (Treasury 1) included the following

items that would alter the relationship between life insurance and alternative investments:

1. lower the maximum tax rate to 35 percent;
2. tax the cash value buildup on life insurance policies currently;
3. eliminate long term capital gains tax treatment, but tax only the excess returns over inflation.

President Reagan's tax proposal [11], released in May, 1985, would also affect the taxation of life insurance and alternative investments. Specifically, this plan would:

1. lower the maximum tax rate to 35 percent;
2. tax the cash value buildup on newly issued life insurance policies currently;
3. tax 50 percent of long term capital gains.

If life insurance policies lose the tax deferment advantage so all interest earned on the cash value is taxed currently, universal/variable life insurance policies would become unmarketable. These policies, with their upfront loads, would never dominate alternative investment strategies. Lowering the maximum tax rate and altering capital gains tax treatment will influence life insurance sales, but it is likely that universal/variable life insurance policies could dominate alternative investment strategies if the holding period is long enough.

Conclusions

Universal/variable life insurance policies allow an investor to participate in the returns of a selected investment mode through a life insurance policy. Tax advantages inherent in life insurance create the situation that purchase of these policies, despite paying

expense loadings above those in comparable investments, is the preferred choice if the policy is held long enough. The necessary holding period depends on a number of values, some known to the policyholder, age, cost of insurance, tax rate, and expense loading, and some unknown, rate of return to be earned through the insurance policy and the alternative investment and the tax status of stock investment earnings. This analysis provides both a method for determining the preferred investment and illustrates the necessary holding period for the universal/variable life policy to dominate under a variety of parameter values. For typical values, the universal/variable life insurance policy dominates the alternative investment strategy in three to six years. A policyholder can estimate the likelihood of keeping the policy in force for the necessary holding period and decide which investment is preferable.

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Footnotes

[1]Since t is restricted to the range 0 to .5, the derivative cannot be negative for $P \leq P'$ and positive for $P > P'$.

Table 1

Total After-Tax Value of a Money Market Savings Fund
or Cash Value for a Universal/Variable Life Policy

Rate of return	r = 10%
Tax rate	t = 40%
Expense loading	e = 6%
Interest rate differential	d = 0%
Annual investment	P = 1000
Face value of policy	F = 100000
Index of competitiveness	g = 1.0
Age of insured	x = 35

Period	$BTID_M$	UVL	Difference
1	948.29	918.08	-30.21
2	1,946.55	1,920.77	-25.78
3	2,996.14	3,008.91	12.77
4	4,099.34	4,125.16	25.82
5	5,257.30	5,305.93	48.63
6	6,472.52	6,557.16	84.64
7	7,746.38	7,884.63	138.25
8	9,082.40	9,295.97	213.57
9	10,482.27	10,798.28	316.01
10	11,949.01	12,400.16	451.15
11	13,484.60	14,110.30	846.50
12	15,092.75	15,939.27	846.52
13	16,776.19	17,897.93	1,121.74
14	18,538.63	19,998.75	1,460.12
15	20,381.93	22,254.18	1,872.25
16	22,309.34	24,678.64	2,369.30
17	24,321.00	27,286.01	2,965.01
18	26,418.30	30,092.27	3,673.97
19	28,601.89	33,114.55	4,512.66
20	30,871.24	36,370.88	5,499.64
21	33,228.64	39,882.88	6,654.24
22	35,674.89	43,673.35	7,998.46
23	38,213.29	47,768.82	9,555.53
24	40,845.70	52,197.55	11,351.85
25	43,573.67	56,990.33	13,416.66

Table 2

Total After-Tax Value of a Stock Market
Savings Fund or Cash Value for a Universal/Variable Life Policy

Rate of return	r = 15%
Tax rate	t = 40%
Short term taxable portion	s = 50%
Long term taxable portion	ℓ = 30%
Expense loading	e = 6%
Interest rate differential	d = 0%
Annual investment	p = 1000
Face value of policy	F = 100000
Index of competitiveness	g = 1.0
Age of insured	x = 35

Period	$BTID_S$	UVL	Difference
1	991.23	959.81	-31.42
2	2,082.74	2,033.64	-49.10
3	3,284.16	3,184.49	-99.67
4	4,607.07	4,441.86	-165.21
5	6,063.09	5,820.40	-242.69
6	7,666.43	7,337.76	-328.67
7	9,431.65	9,013.44	-418.21
8	11,377.05	10,871.19	-505.86
9	13,520.90	12,936.98	-583.92
10	15,884.81	15,241.49	-643.32
11	18,491.55	17,819.20	-672.35
12	21,368.17	20,710.84	-657.33
13	24,543.48	23,962.42	-581.06
14	28,050.44	27,627.40	-423.04
15	31,923.57	31,765.95	-157.62
16	36,202.57	36,448.03	245.46
17	40,928.24	41,751.98	823.74
18	46,147.27	47,768.71	1,621.44
19	51,910.78	54,602.20	2,691.42
20	58,274.40	62,371.24	4,096.84
21	65,303.05	71,214.35	5,911.30
22	73,067.26	81,289.68	8,222.42
23	81,648.13	92,780.74	11,132.61
24	91,134.16	105,897.50	14,763.34
25	101,623.50	120,881.20	19,257.70

Table 3

Standard Values

Rate of return (%)	(r)	10 for money market fund 15 for stock fund
Index of competitiveness	(g)	1.0
Tax rate (%)	(t)	40
Expense rate (%)	(e)	6
Interest rate differential (%)	(d)	0
Annual investment ($)	(P)	1,000
Face value of policy ($)	(F)	100,000
Age of insured	(x)	35
Short term gains realized (%)	(s)	50
Long term gains realized (%)	(ℓ)	30

Table 4

Effect of Varying Rate of Return and Annual Investment on Year Universal/Variable Life Dominates BTID

Rate of Return (%)	Annual Investment													
	250		500		1,000		2,500		5,000		10,000		25,000	
	M	S	M	S	M	S	M	S	M	S	M	S	M	S
4	IP	IP	10	15	8	17	16	ND	21	PAR	PAR	PAR	PAR	PAR
6	10	IP	6	9	5	22	13	ND	16	PAR	17	PAR	PAR	PAR
8	7	11	5	7	4	22	11	28	12	PAR	13	PAR	PAR	PAR
10	6	9	4	6	3	20	9	24	10	PAR	11	PAR	PAR	PAR
12	5	7	3	5	3	18	8	21	9	PAR	9	PAR	PAR	PAR
14	4	6	3	4	2	17	7	18	8	PAR	8	PAR	PAR	PAR
15	4	5	2	4	2	16	7	17	7	PAR	8	PAR	PAR	PAR
16	3	5	2	3	2	15	6	16	7	PAR	7	PAR	PAR	PAR
18	3	4	2	3	2	14	6	15	6	PAR	7	PAR	PAR	PAR
20	3	4	2	3	2	13	5	14	6	PAR	6	PAR	6	PAR

M = money market fund investment option
S = stock market fund investment option
IP = insufficient premium
ND = never dominates
PAR = policy adjustment required

Table 5

Effect of Varying Expense Loading and Annual Investment on Year Universal/Variable Life Dominates BTID

Expense Loading(%)	Annual Investment 250		500		1,000		2,500		5,000		10,000		25,000	
	M	S	M	S	M	S	M	S	M	S	M	S	M	S
2	1	1	1	1	1	1	1	14	4	PAR	6	PAR	6	PAR
4	3	3	2	2	2	14	6	16	8	PAR	9	PAR	PAR	PAR
6	6	5	4	4	3	16	9	17	10	PAR	11	PAR	PAR	PAR
8	8	8	5	14	7	18	11	19	12	PAR	13	PAR	PAR	PAR
10	11	10	7	17	10	19	13	20	14	PAR	14	PAR	PAR	PAR
12	14	13	8	19	13	21	15	PAR	16	PAR	PAR	PAR	PAR	PAR
14	22	16	10	21	15	22	17	PAR	17	PAR	PAR	PAR	PAR	PAR
16	IP	21	11	23	17	23	18	PAR	19	PAR	PAR	PAR	PAR	PAR
18	IP	ND	14	25	18	25	20	PAR	20	PAR	PAR	PAR	PAR	PAR
20	IP	ND	17	27	20	26	21	PAR	22	PAR	PAR	PAR	PAR	PAR

M = money market fund investment option
S = stock market fund investment option
IP = insufficient premium
ND = never dominates
PAR = policy adjustment required

Table 6

Effect of Varying Tax Rate and Annual Investment on Year Universal/Variable Life Dominates BTID

Tax Rate (%)	Annual Investment													
	250		500		1,000		2,500		5,000		10,000		25,000	
	M	S	M	S	M	S	M	S	M	S	M	S	M	S
0	IP	ND	ND	ND	ND	PAR	PAR	PAR	PAR	PAR	PAR	PAR	PAR	PAR
5	IP	ND	ND	ND	ND	PAR	PAR	PAR	PAR	PAR	PAR	PAR	PAR	PAR
10	IP	ND	22	27	22	25	22	PAR	PAR	PAR	PAR	PAR	PAR	PAR
15	19	22	15	21	17	21	18	PAR	18	PAR	PAR	PAR	PAR	PAR
20	12	19	9	18	14	19	15	19	15	PAR	PAR	PAR	PAR	PAR
25	10	9	6	16	11	18	13	18	14	PAR	14	PAR	PAR	PAR
30	8	7	5	14	8	17	12	18	12	PAR	13	PAR	PAR	PAR
35	6	6	4	12	6	16	10	17	11	PAR	12	PAR	PAR	PAR
40	6	5	4	4	3	16	9	17	10	PAR	11	PAR	PAR	PAR
45	5	5	3	3	3	16	8	17	9	PAR	10	PAR	PAR	PAR
50	4	4	3	3	2	16	7	18	9	PAR	9	PAR	PAR	PAR

M = money market fund investment option
S = stock market fund investment option
IP = insufficient premium
ND = never dominates
PAR = policy adjustment required

Table 7

Effect of Varying Index of Competitiveness and Annual Investement on Year Universal/Variable Life Dominates BTID

Index of Competitiveness	Annual Investment													
	250		500		1,000		2,500		5,000		10,000		25,000	
	M	S	M	S	M	S	M	S	M	S	M	S	M	S
0.6	1	1	1	1	1	1	6	16	9	PAR	10	PAR	PAR	PAR
0.7	1	1	1	1	1	1	7	16	9	PAR	10	PAR	PAR	PAR
0.8	1	1	1	1	2	13	8	16	10	PAR	11	PAR	PAR	PAR
0.9	1	1	2	2	3	14	8	17	10	PAR	11	PAR	PAR	PAR
1.0	6	5	4	4	3	16	9	17	10	PAR	11	PAR	PAR	PAR
1.1	IP	13	5	16	5	17	10	18	11	PAR	11	PAR	PAR	PAR
1.2	IP	IP	7	20	8	19	10	18	11	PAR	11	PAR	PAR	PAR
1.3	IP	IP	10	24	10	20	11	19	11	PAR	11	PAR	PAR	PAR
1.4	IP	IP	16	28	13	21	12	19	11	PAR	11	PAR	PAR	PAR

M = money market fund investment option
S = stock market fund investment option
IP = insufficient premium
PAR = policy adjustment required

Table 8

Effect of Varying Age and Annual Investment on
Year Universal/Variable Life Dominates BTID

Starting Age	Annual Investment													
	250		500		1,000		2,500		5,000		10,000		25,000	
	M	S	M	S	M	S	M	S	M	S	M	S	M	S
20	5	5	4	17	6	17	10	18	11	PAR	11	PAR	PAR	PAR
25	5	4	3	16	7	17	10	18	11	PAR	11	PAR	PAR	PAR
30	5	5	3	15	6	17	10	18	11	PAR	11	PAR	PAR	PAR
35	6	5	4	4	3	16	9	17	10	PAR	11	PAR	PAR	PAR
40	IP	IP	4	4	3	14	8	17	10	PAR	11	PAR	PAR	PAR
45	IP	IP	6	6	4	4	4	16	9	PAR	10	PAR	PAR	PAR
50	IP	IP	IP	IP	5	5	3	15	7	PAR	9	PAR	PAR	PAR
55	IP	IP	IP	IP	8	8	4	4	3	16	8	PAR	PAR	PAR
60	IP	IP	IP	IP	IP	IP	5	4	3	13	5	PAR	PAR	PAR
65	IP	IP	IP	IP	IP	IP	8	7	4	4	3	PAR	8	PAR

M = money market fund investment option
S = stock market fund investment option
IP = insufficient premium
PAR = policy adjustment required

Table 9

Effect of Varying Interest Rate Differential and Annual Investment on Year Universal/Variable Life Dominates BTID

Interest Rate Difference (%)	Annual Investment													
	250		500		1,000		2,500		5,000		10,000		25,000	
	M	S	M	S	M	S	M	S	M	S	M	S	M	S
-4	IP	ND	ND	ND	ND	ND	ND	PAR	PAR	PAR	PAR	PAR	PAR	PAR
-3	IP	ND	27	ND	ND	ND	ND	PAR	PAR	PAR	PAR	PAR	PAR	PAR
-2	13	12	8	ND	26	ND	ND	PAR	PAR	PAR	PAR	PAR	PAR	PAR
-1	8	7	5	21	11	24	17	PAR	18	PAR	PAR	PAR	PAR	PAR
0	6	5	4	4	3	16	9	17	10	PAR	11	PAR	PAR	PAR
1	4	4	3	3	2	10	5	12	6	13	7	PAR	7	PAR
2	3	3	2	2	2	6	3	9	4	9	5	9	5	PAR
3	3	3	2	2	2	2	2	6	3	7	3	7	3	PAR
4	3	3	2	2	1	1	2	4	2	5	2	5	3	5

M = money market fund investment option
S = stock market fund investment option
IP = insufficient premium
ND = never dominates
PAR = policy adjustment required

Table 10

Effect of Varying Face Value and Annual Investment on Year Universal/Variable Life Dominates BTID

Face Value	Annual Investment													
	250		500		1,000		2,500		5,000		10,000		25,000	
	M	S	M	S	M	S	M	S	M	S	M	S	M	S
25,000	3	16	8	17	10	PAR	11	PAR	PAR	PAR	PAR	PAR	PAR	PAR
50,000	4	4	3	16	8	17	10	PAR	11	PAR	PAR	PAR	PAR	PAR
100,000	6	5	4	4	3	16	9	17	10	PAR	11	PAR	PAR	PAR
250,000	IP	IP	8	7	4	4	3	16	8	17	10	PAR	11	PAR
500,000	IP	IP	IP	IP	8	7	4	4	3	16	8	17	10	17
1,000,000	IP	IP	IP	IP	IP	IP	6	5	4	4	3	16	9	16
2,500,000	IP	IP	IP	IP	IP	IP	IP	IP	8	7	4	4	3	4
5,000,000	IP	IP	IP	IP	IP	IP	IP	IP	IP	IP	8	7	4	7

M = money market fund investment option
S = stock market fund investment option
IP = insufficient premium
PAR = policy adjustment required

Table 11

Effect of Varying Tax Allocation for Stock Fund and Annual Investment on Year Universal/Variable Life Dominates BTID

Percentage of Gains Taxable as		Annual Investment						
Short Term	Long Term	250	500	1,000	2,500	5,000	10,000	25,000
0	0	ND	ND	PAR	PAR	PAR	PAR	PAR
0	100	10	30	PAR	PAR	PAR	PAR	PAR
20	30	8	ND	PAR	PAR	PAR	PAR	PAR
50	30	5	4	16	17	PAR	PAR	PAR
100	0	4	2	2	7	7	8	PAR

ND = never dominates
PAR = policy adjustment required

Table 12

Optimal Annual Investment in Universal/Variable Life Policy for Standard Parameters with One Variable Changing

Variable g	Year	P^*_M	Year	P^*_S	Variable d	Year	P^*_M	Year	P^*_S
.6	1	106[a]	1	106[a]	-2	7	602	9	319
.8	1	106[a]	1	106[a]	0	3	965	4	473
1.0	3	965	4	473	+2	2	1110	2	712
1.2	6	637	18	3259[b]	+4	1	1678	1	1057
1.4	11	15880[b]	19	2870[b]					

Variable r	Year	P^*_M	Year	P^*_S	Variable e	Year	P^*_M	Year	P^*_S
6	5	1164	8	756	0	1	1159	1	808
8	4	980	6	679	3	1	1730	1	1049
10	3	965	5	611	6	3	965	4	473
12	3	765	4	599	9	5	722	6	390
15	2	827	4	473	12	8	542	10	305
18	2	667	3	489					
20	2	594	3	440					

Variable t	Year	P^*_M	Year	P^*_S
20	8	435	12	250
30	5	612	5	402
40	3	965	4	473
50	2	1462	3	592

[a] Minimum amount possible to avoid insufficient premium
[b] Maximum amount allowed without policy adjustment requirement